

Block Spin Effective Action for Polyakov Loops in 4D SU(2) LGT *

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Using a variant of the IMCRG method of Gupta and Cordery, we explicitly compute majority rule block spin effective actions for the signs of the Polyakov loops in 4D SU(2) finite temperature lattice gauge theories. To the best of our knowledge, this is the first attempt to compute numerically effective actions for the Polyakov loop degrees of freedom in 4D SU(2). The most important observations are: 1. The renormalization group flow at the deconfinement transition can be nicely matched with the flow of the 3D Ising model, thus confirming the Svetitsky-Yaffe conjecture. 2. The IMCRG simulations of the FT SU(2) model have strongly reduced critical slowing down.

1. INTRODUCTION

D+1-dimensional pure LGT's with finite periodic extension N_t in the “time” direction describe glue systems at finite temperature $1/N_t$. They undergo a deconfinement transition, signalled by a finite expectation value of the trace of the Polyakov loop.

According to the 15 years old Svetitsky-Yaffe conjecture [1], the D-dimensional effective statistical model for the Polyakov loops has an action (Hamiltonian) with short range interactions and a global symmetry group given by the center of SU(N). If both the deconfining phase transition of the gauge model and of the corresponding order-disorder phase transition of the spin system are continuous, the two models belong to the same universality class.

The conjecture that the deconfinement transition of 4D SU(2) LGT belongs to the 3D Ising universality class is supported by analytical calculations, and numerical estimates of its critical indices, c.f. [2].

We tested the Svetitsky-Yaffe conjecture by comparing flows of block spin effective actions for the Polyakov loops with flows of the 3D Ising model. Approach of both flows to a single trajectory ending in a common renormalization group fixed point demonstrates universality on a fundamental level.

2. BLOCKING POLYAKOV LOOPS

Our procedure to generate and analyse block spin effective actions for Polyakov loops consists of the following steps.

- Map the SU(2) configurations U living on an $N_t \times N_s^3$ lattice to Ising configurations $\sigma(U)$ on an N_s^3 lattice. The Ising variables are given by the signs of the Polyakov loops.
- Block the Ising configurations with the majority rule, using cubical blocks of size L_B .
- Compute the effective coupling constants using IMCRG [3].
- Generate a renormalization group flow by increasing the block size L_B .
- Compare the resulting flow with that computed directly in the Ising model.

More formally, the effective action for the signs of the Polyakov loops is given by

$$\exp[-H'(\mu)] = \int DU P(\mu, U) \exp[-S_g(U)], \quad (1)$$

with

$$P(\mu, U) = \prod_{x'}^{(\text{blocks})} \frac{1}{2} \left[1 + \mu_{x'} \text{sign} \sum_{x \in x'} \sigma_x(U) \right]. \quad (2)$$

Here, S_g is the standard Wilson action for SU(2). In case of an even block size L_B the sum of the

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Ising spins σ_x inside a block x' can be zero. In that case a positive (negative) $\mu_{x'}$ is selected with probability one half.

Unfortunately, effective Hamiltonians contain an infinite number of coupling constants. In practical calculations one has to truncate to a finite set of interactions. We chose to include in the ansatz eight 2-point couplings and six 4-point couplings. The 2-point couplings can be labelled by specifying the relative position of the interacting spins (up to obvious symmetries): our couplings $K_1 \dots K_8$ then correspond to 001,011,111,002,012,112,022,122. The 4-point couplings $K_9 \dots K_{14}$ are defined through Fig. 1. The corresponding interaction terms in the effective Hamiltonian are denoted by S'_α , $\alpha = 1 \dots 14$.

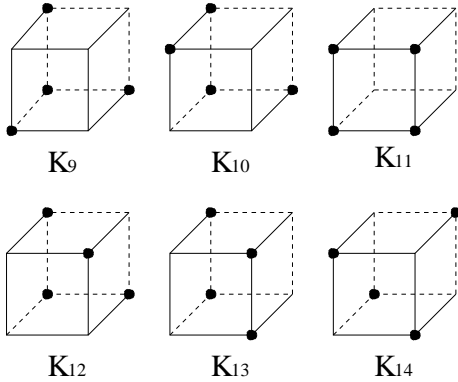


Fig. 1. Definition of the 4-point-couplings.

3. IMCRG FOR POLYAKOV LOOPS

Translating the rationale of IMCRG [3] to the present context means that one does *not* simulate standard SU(2) model but instead the system with partition function

$$\sum_{\mu} \int DU P(\mu, U) \exp [-S_g(U) + \bar{H}(\mu)] , \quad (3)$$

where $\bar{H}(\mu) = \sum_{\alpha} \bar{K}_{\alpha} S'_{\alpha}(\mu)$ is a guess for $H'(\mu)$. Note the *plus* sign in front of \bar{H} . As before, μ denote the block spins, defined on blocks of size L_B obtained from blocking the Polyakov loops. The crucial observation is now that if $\bar{H} = H'$, i.e., if the guess of the Hamiltonian is right, then

the $\mu_{x'}$ decouple completely. This means in particular that the correlations $\langle S'_{\alpha}(\mu) \rangle$ vanish. A non-perfect guess can be improved by iteration of

$$\bar{K}_{\alpha} \longrightarrow \bar{K}_{\alpha} + n_{\alpha}^{-1} \langle S'_{\alpha}(\mu) \rangle , \quad (4)$$

where n_{α} are trivial multiplicity factors. A few iterations of this correction step usually suffice to obtain good precision for the effective couplings.

This method, though it seems to be restricted to Ising type models, has several merits. One of them is the following: For small enough blocks, the autocorrelations in the simulations are drastically reduced. This follows from the fact that in case of perfect compensation the blocks completely decouple and fluctuate independently. In the SU(2) IMCRG calculations discussed below this phenomenon was clearly observed [4].

4. MONTE CARLO RESULTS

We started by computing the flow of the critical 3D Ising model, using two different models. The standard Ising model with nearest neighbour couplings becomes critical at $\beta = 0.2216544$. A version “ I_3 ” which includes also third (cube-diagonal) neighbour couplings has a critical point at $\beta_1 = 0.128003$ and $\beta_3 = 0.051201$ [5].

We then turned to FT SU(2) with $N_t = 2$. We computed the effective actions from simulations on lattices consisting of 6^3 blocks of size $L_B \leq 6$. The simulations were performed at $\beta_g = 1.880, 1.877, 1.874, 1.871$. Best matching with the Ising data was achieved at $\beta_g = 1.877$. The flow of K_1 and K_2 for this gauge coupling is shown in Fig. 2, together with the Ising flows. The gauge data are displayed with squares, the Ising results are shown with bars, diamonds and dotted fit lines. The fits were done with a simple power law [4]. A nice matching is observed also for the 12 couplings not shown here. Note that the block sizes of the various models have to be rescaled with respect to each other in order to yield matching. E.g., the L_B from the I_3 model have to be rescaled by a factor of 0.59 with respect to that of the standard Ising model.

Let us finally show a comparison of the flow of the nearest neighbour coupling for different gauge couplings. This is shown in Fig. 3. The dashed

line, together with diamonds and crosses, gives the Ising flow. The other data (from top to bottom) correspond to $\beta_g = 1.880$ (triangles), 1.877 (crosses), 1.774 (squares), and 1.771 (stars).

The gauge block sizes are rescaled such that best matching is obtained for $\beta_g = 1.877$. Within the given precision, also the $\beta_g = 1.874$ data could be rescaled to match the Ising flow. This is, however, clearly ruled out for $\beta_g = 1.880$.

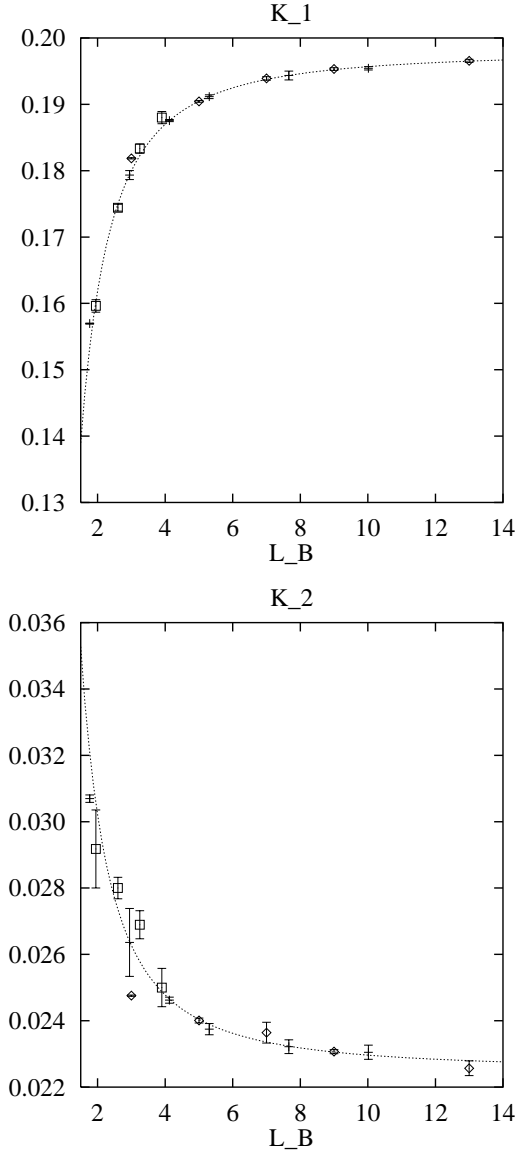


Fig. 2. Matching of the couplings K_1 and K_2 .

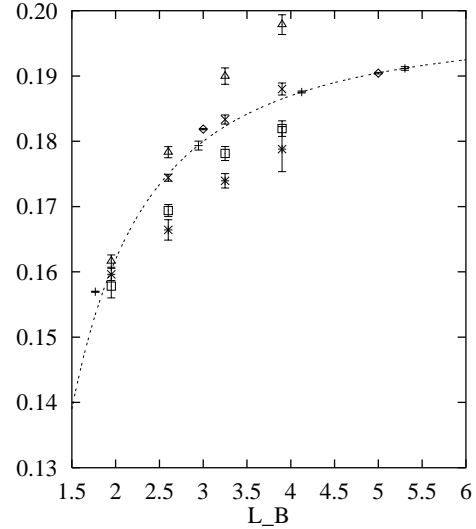


Fig. 3. Comparison of K_1 flow for different β_g .

5. CONCLUSIONS

IMCRG works well as a method to compute the effective action of Ising type degrees of freedom, also in non-Ising models like 4D FT SU(2) LGT. The calculations could be done on workstations. The Svetitsky-Yaffe conjecture is confirmed in a very fundamental way by observing matching of the RG trajectories with those of the Ising model. We obtained results for $N_t = 1$ also, see [4]. Extension to N_t bigger than two is expensive, because one needs larger blocks to come close enough to the fixed point.

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